Soft phonon mode as the origin of the reduced thermal conductivity in Ta-based trirutiles

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Ta-based trirutiles of the series $A\mathrm{Ta_2O_6}$ (A = Ni, Co) reportedly present suppressed thermal conductivity (κ) values compared to their Sb-based counterparts. Particularly, the κ values at room temperature for Ta-based samples are in the range observed in materials already employed as thermoelectric devices, suggesting they are potential candidates as a starting point for thermoelectric applications. Here, we investigate their phonon dynamics through a combination of Raman scattering measurements with density functional theory (DFT) calculations. For the Ta-based compounds only, our results reveal the presence of an E_g low-energy optical phonon that softens by ~ 10 cm^{-1} upon cooling from 300 K to 15 K, indicating this is a zone-center soft mode associated with an unrealized structural phase transition. The soft mode enhances the phonon density of states at low energies, as directly manifested in the second-order Raman scattering data and also captured by DFT phonon calculations. These results provide insights into the low κ-values of Ta-based trirutiles and places zone-center soft phonons as a key ingredient for the development of novel thermoelectic materials.

Thermoelectric materials can convert heat, including waste heat, into electricity and vice-versa without greenhouse emissions, making them proper candidates for sustainable energy solutions[1,2]. Their efficiency in energy conversion is quantified by the dimensionless figure of merit, $ZT = \sigma S^2 T/\kappa$, where σ is the electrical conductivity, S is the Seebeck coefficient, T is the absolute temperature, and κ is the thermal conductivity. The main drawback concerning their applicability is the complex interplay between σ, S, and κ, which imposes significant difficulties when attempting to increase ZT[3,4]. In this regard, a promising strategy is to explore compounds that have intrinsically low thermal conductivity and try to enhance the other two parameters[1–4].

In the range of temperatures where thermoelectric materials are employed, phonons are the primary heat carriers[5,6]. Therefore, compounds where phonons are strongly scattered are potential candidates for application as thermoelectric devices[7]. In this context, three classes of compounds call for attention: the clathrates, skutterudites, and IV-VI chalcogenides[8]. For all of them, the observed low thermal conductivity originates from the unusual presence of a low-energy optical phonon mode and its interaction with the acoustic branch[9–20].

In this context, another noteworthy class of compounds is the family $AB_2\mathrm{O_6}$ (A = Co, Ni, Cu; B = Sb, Ta), crystallizing in the trirutile structure with the tetragonal space group $P4_2/mnm$[21–25]. Much of the interest in these systems concerns their magnetic properties. For all of them, one-dimensional spin chains of transition metal ions exhibit long-range antiferromagnetic ordering, with short-range correlations starting well above T_N[21–25], offering a rich platform to study magnetic interactions in reduced dimensions. Besides those appealing magnetic properties, these compounds also display an intricate behavior in their thermal properties[26]. Compounds with Ta exhibit κ values at room temperature comparable to those found in materials already used as thermoelectric devices[2,3,26]. In addition, comparing the thermal conductivity between Ta- and Sb-based trirutiles, a suppression of about one order of magnitude is observed for the samples with Ta, regardless of the transition metal ion at the A site[26]. Employing the Callaway model and considering the scattering of acoustic phonons by point defects, dislocations, boundaries, and Umklapp processes is sufficient to reproduce the thermal conductivity of $\mathrm{NiSb_2O_6}$[26]. For $\mathrm{NiTa_2O_6}$, besides those processes, an additional scattering mechanism from optical phonons must be invoked[26]. Thus, a possible proximity of an optical phonon with the acoustic branch remains elusive and calls for an experimental verification.

In this work, we investigate the origin of the suppressed thermal conductivity in Ta-based trirutiles with respect to their Sb counterparts in the $AB_2\mathrm{O_6}$ family (A = Ni, Co, Zn; B = Sb, Ta) by combining temperature-dependent Raman spectroscopy measurements in the paramagnetic phase with density functional theory (DFT) calculations. We found that the Ta-based samples have a low-energy optical phonon that softens throughout the entire measured temperature range, suggesting an incipient soft-mode driven structural instability. The presence of this optical mode provides additional decay channels for the heat-carrying acoustic phonons, yielding insights into the microscopic origin of the thermal properties of these compounds.

Figures 1(a) and 1(b) exhibit the low-frequency region of the calculated phonon dispersions for $\mathrm{NiTa_2O_6}$ and $\mathrm{NiSb_2O_6}$, (for more details about the DFT calculations, see Section S1 in the Supplementary Material), respectively, highlighting the

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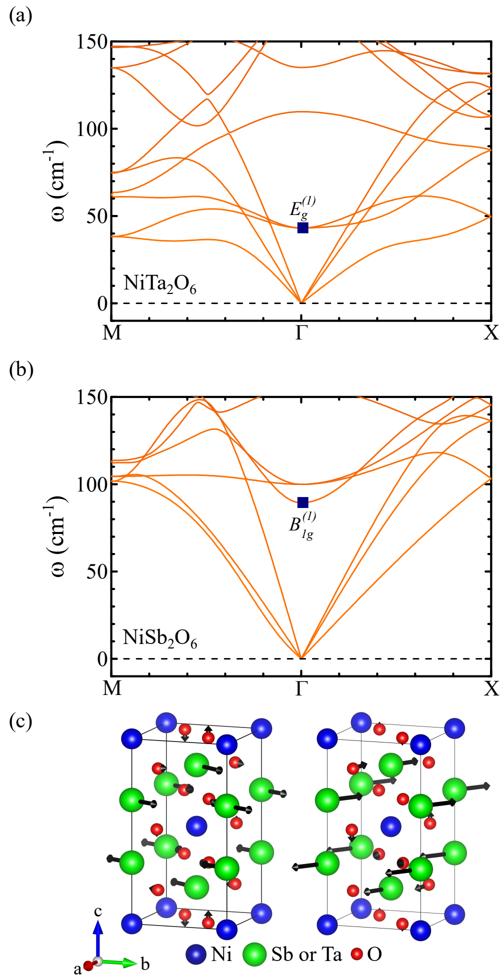

FIG. 1. Phonon dispersion relation along $M-\Gamma-X$ directions of the Brillouin zone for (a) $NiTa_2O_6$ and (b) $NiSb_2O_6$ only showing the low-frequency region. Blue squares indicate the lowest optical phonon mode in both panels. (c) Mechanical representation of the doubly-degenerate $E_g^{(1)}$ mode, with black arrows indicating the direction of the vibration.

behavior of the acoustic and the less energetic optical phonon branches. Remarkably, $NiTa_2O_6$ presents a low-frequency optical phonon at the Γ point (calculated $\omega = 43$ cm^{-1}), which is the $E_g^{(1)}$ Raman-active doubly degenerate mode [see Table S2 in the Supplementary Material and Fig. 1(c) for its mechanical representation]. $NiSb_2O_6$, on the other hand, displays its lowest-frequency optical mode at 89 cm^{-1}, which is a nondegenerate B_{1g} mode, while the $E_g^{(1)}$ mode occurs at 100 cm^{-1}. Close to the Γ-point, the dispersions of the three acoustic branches are similar for both compounds. However, at the M and X points, the acoustic modes for the Ta-based compound are softer by a factor of two or more with respect to $NiSb_2O_6$.

We proceed to the theoretical ph-DOS, shown in Figs. 2(a) and 2(b). For $NiTa_2O_6$ [Fig. 2(a)], a substantial ph-DOS is noticed for 2ω between ~ 60 and 140 cm^{-1}, which originates from both the low-frequency optical and the acoustic branches [Fig. 1(a)]. For $NiSb_2O_6$ [Fig. 2(b)], the ph-DOS remains quite small for $2\omega < 150$ cm^{-1}, as could be anticipated from the absence of an optical mode in this frequency region and the much wider dispersion of the acoustic branches in comparison with the Ta-based compound [Fig. 1(b)].

Figures 2(c) and 2(d) display the experimental low-ω Raman response of $NiTa_2O_6$ and $NiSb_2O_6$ (for more details about the sample growth methods and Raman spectroscopy measurements, see Sections S2 and S3, respectively, in the Supplementary Material), respectively, at $T = 15$ K. For $NiTa_2O_6$, two first-order modes are observed at $\omega = 29$ cm^{-1} and 128 cm^{-1} and assigned as $E_g^{(1)}$ and $B_{1g}^{(1)}$ modes, respectively (see Table S2 in the Supplementary Material). A broad scattering is also detected for ω between 50 and 120 cm^{-1}, characteristic of second-order Raman process. In addition, the shape of this signal shows a close correspondence with the calculated ph-DOS in Fig. 2(a), supporting its assignment to two-phonon Raman scattering. This method is a well-known probe of the ph-DOS[28? –30].

The thermal evolution of the low-ω Raman signal of $NiTa_2O_6$ is presented in Fig. 3(a). Remarkably, the $E_g^{(1)}$ mode shows a substantial hardening from 29 cm^{-1} at $T = 15$ K to 39 cm^{-1} at $T = 290$ K. This is the opposite trend of conventional behavior, where the volume expansion associated with the cubic anharmonic term of the interatomic potential promotes a slight softening of the vibrations upon warming[31–33]. In the present case, the strong temperature dependence of the $E_g^{(1)}$ frequency is indicative of a soft mode behavior that occurs in the vicinity of displacive structural phase transitions[34–36], where the soft mode frequency behaves as an order parameter of the transition and follows Cochran's equation $\omega \propto (T-T_c)^{1/2}$ (Ref. 27). Using this expression to fit our data [see Fig. 3(c)], we obtain $T_c = -341$ K for $NiTa_2O_6$. This negative value for T_c indicates an unrealized structural phase transition. The other investigated Ta-based compound, $CoTa_2O_6$, presents similar results, except that the $E_g^{(1)}$ mode is harder in this case, indicating that this compound is farther away from the structural phase transition with $T_c = -471$ K [see Figs. 3(b) and 3(c)].

Further information about the crystal structure dynamics of these materials is obtained by examining the high-ω modes, which are associated with oxygen vibrations. The symmetry analysis and phonon assignments are provided in the Supplementary Material (see Tables S1 and S2). Figures 4(a) - 4(d) display the Raman response at selected temperatures for $NiTa_2O_6$, $CoTa_2O_6$, $NiSb_2O_6$, and $ZnSb_2O_6$, respectively. It is possible to notice that the high-ω modes of ATa_2O_6 (A =

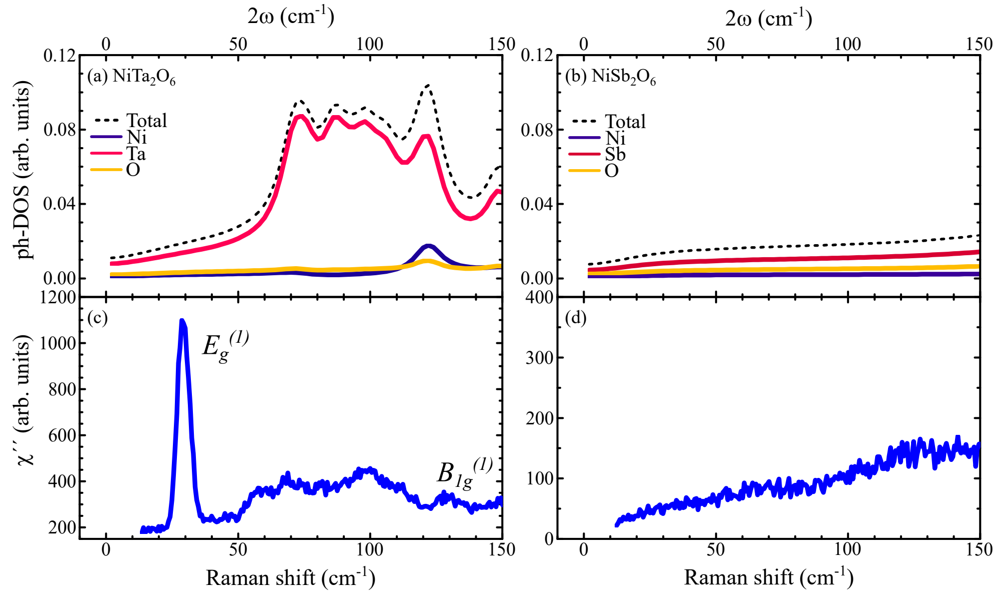

FIG. 2. Calculated phonon density of states for (a) $NiTa_2O_6$ and (b) $NiSb_2O_6$. Raman response measured at 15 K for (c) $NiTa_2O_6$ and (d) $NiSb_2O_6$.

Ni, Co) show a more pronounced broadening upon warming than the Sb-based compounds. We investigate in more detail the $A_{1g}^{(4)}$ mode at $\omega \sim 680-750$ cm^{-1}, highlighted with an asterisk in Figs. 4(a) - 4(d). The mechanical representation of this particular mode is indicated in Fig. 5(a). Figures 5(b) - 5(e) show the temperature-dependence of its linewidth (Γ) for the compounds studied in this work. For Sb-based compounds, the phonon broadening is well captured by a conventional two-phonon decay model $\Gamma(T) = \Gamma_0[1+2/(e^x-1)]$, where $x \equiv \hbar\omega_0/k_bT$ (refs. 32 and 37). However, for Ta-based compounds, the thermal broadening effect is much larger than predicted by this model at $T \gtrsim 30$ K. Although this is a simplified model, this comparison clearly indicates that an additional source of anharmonicity, not present in the investigated Sb-based compounds, is responsible for the reduced lifetime of the high-frequency optical modes for the Ta-based materials.

In ordered magnetic materials, phonon frequencies can also be renormalized through spin-phonon coupling at characteristic temperatures associated with the onset of long-range magnetic ordering[38] or short-range magnetic correlations[39,40]. This raises the question of whether spin degrees of freedom contribute to the temperature dependence observed for the E_g mode. All the Raman measurements in this study were conducted above the Néel temperature of the compounds[21–25]. In addition, short-range correlations in all cases extend up to 30 K[21–25]. Since the E_g mode softens across the entire temperature range, with no apparent change at those temperatures, the spin-phonon coupling is not significant for the underlying physics of the E_g mode. Instead, its behavior is of anharmonic origin.

The pronounced temperature-induced renormalization of the $E_g^{(1)}$ phonon frequency implies a strongly anharmonic interatomic potential as a function of the corresponding normal coordinate in ATa_2O_6 (A = Ni and Co). Its temperature-dependence indicates this is the soft mode of an unrealized symmetry-breaking instability, placing them in the vicinity of a symmetry-lowering structural phase transition. The negative transition temperatures from Cochran's equation[27] found here for $NiTa_2O_6$ ($T_c = -341$ K) and $CoTa_2O_6$ ($T_c = -471$ K) are characteristic of incipient ferroelectric materials. Typical examples are TiO_2[41], with $T_c = -540$ K[42–45], and PbTe, with T_c ranging from -151 K to -79 K[46–48]. In the present case, we stress that the soft mode is a Raman-active E_g mode that is not associated with an oscilating electric dipole. Therefore, the presumed structural transition associated with this mode retains the inversion center and, thus, cannot be associated with a ferroelectric state.

Incipient ordering, as shown here for Ta-based compounds, indicates some mechanism is pushing the material away from the critical point. As mentioned before, this is the case of PbTe, which lies close to a ferroelectric order, and a small amount of Ge doping on the Pb site can induce the ferroelectric state[49]. Therefore, it remains to be determined which kind of ordering could emerge in samples with Ta if the E_g mode softens entirely. In this regard, external pressure is a natural

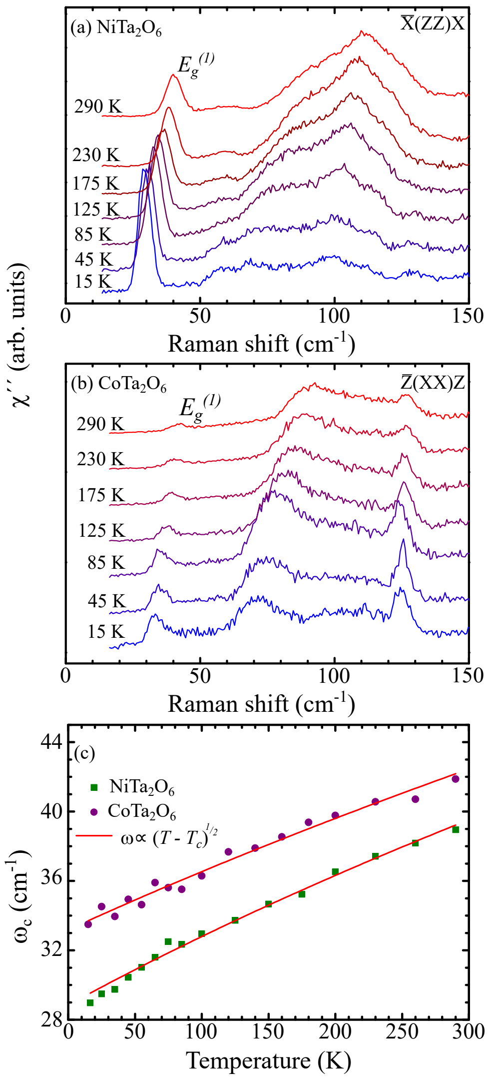

FIG. 3. Raman response of the low-frequency region (ω < 150 cm^{-1}) at selected temperatures for (a) $NiTa_2O_6$ and (b) $CoTa_2O_6$. (c) $E_g^{(1)}$ peak position as a function of temperature for $NiTa_2O_6$ (green squares) and $CoTa_2O_6$ (purple circles). The red lines correspond to fits with the function $\omega \propto (T - T_c)^{\frac{1}{2}}$, where T_c denotes the transition temperature to a lower-symmetry structure[27].

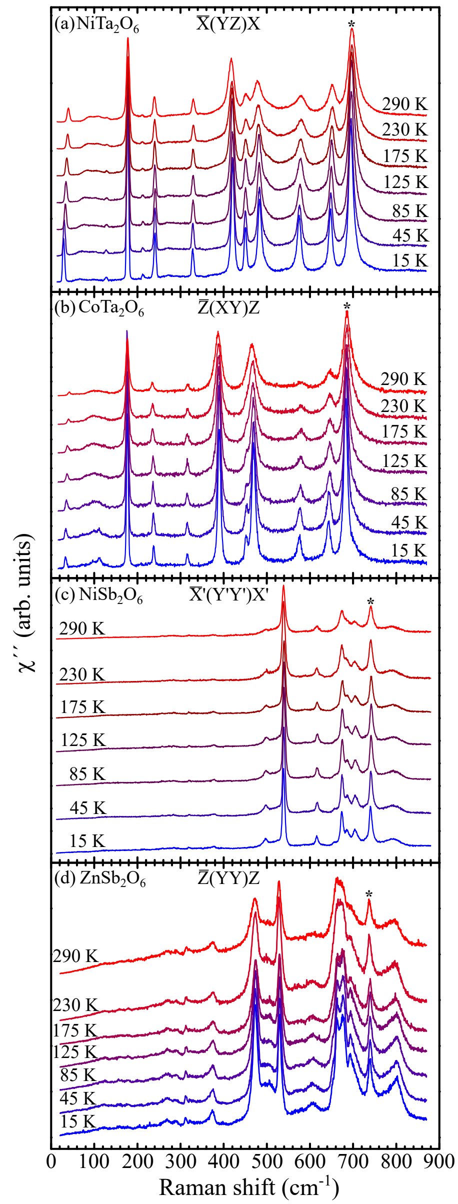

FIG. 4. Raman response at selected temperatures for (a) $NiTa_2O_6$, (b) $CoTa_2O_6$, (c) $NiSb_2O_6$, (d) and $ZnSb_2O_6$. The asterisk in each panel highlights the $A_{1g}^{(4)}$ mode.

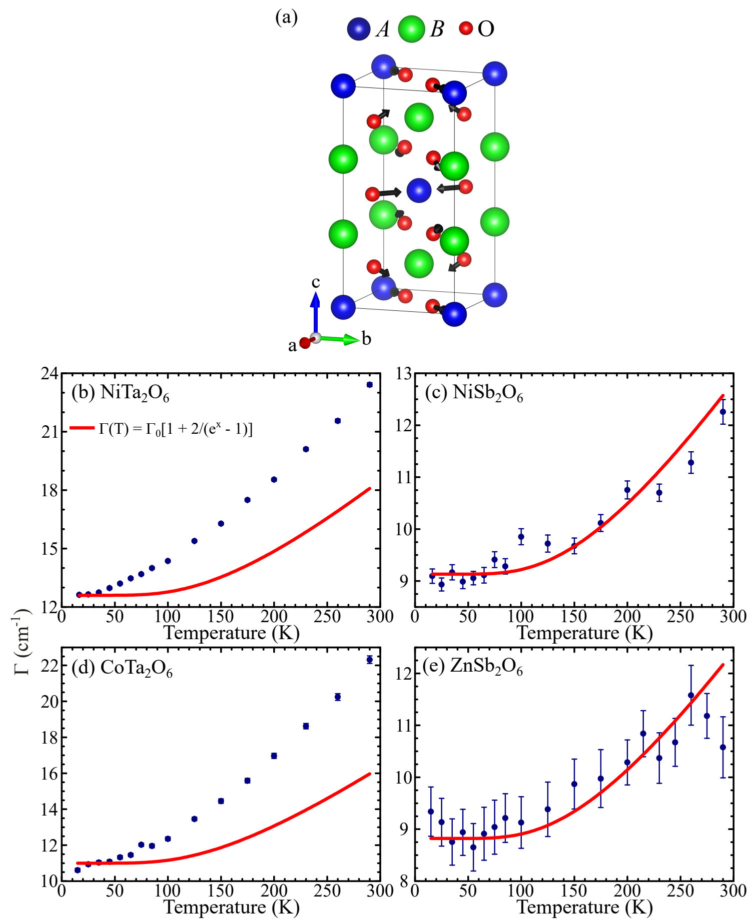

FIG. 5. (a) Mechanical representation of the $A_{1g}^{(4)}$ mode, with black arrows indicating the direction of the vibration. Width (Γ) of the $A_{1g}^{(4)}$ mode as a function of temperature for (b) $NiTa_2O_6$, (c) $NiSb_2O_6$, (d) $CoTa_2O_6$, and (e) $ZnSb_2O_6$. Red curves are fits with the function $\Gamma(T) = \Gamma_0[1+2/(e^x-1)]$, where $x \equiv \hbar\omega_0/k_bT$ (ref. 32).

way to shed light on this scenario. Recent Raman and x-ray diffraction measurements on polycrystalline $NiTa_2O_6$ at room temperature up to 25 GPa revealed a structural phase transition near 12 GPa to a lower-symmetry crystal structure[50]. However, due to experimental constraints, the low-frequency region containing the $E_g^{(1)}$ mode was not probed as a function of pressure. Thus, the pressure dependence of the soft mode remains unknown, and future Raman experiments under pressure at different temperatures are demanded. This will enable the study of whether the structural phase transition is driven by the soft mode and also how pressure influences the anharmonic interactions in Ta-based analogs.

Most materials with noticeably low κ-values have in common one or more low-ω optical phonon branches that interact with the acoustic ones[9–16]. As demonstrated here, this is also the case for the Ta-based trirutiles [see Fig. 1(a)]. In contrast, the lowest optical phonon mode in $NiSb_2O_6$ is nearly twice as high in energy as in its Ta-based counterpart [see Figs. 1(a) and 1(b)]. As a result, the additional scattering mechanism of the heat-carrying acoustic phonons by an optical mode plays a significant role in the thermal conductivity of compounds with Ta, while it might contribute only marginally for Sb-based analogs. This interaction manifests in samples with Ta as a two-phonon Raman signal [see Figs. 2(c) and 2(d)] and is evidenced by its temperature dependence. Since this signal contains contributions from the acoustic branches and the $E_g^{(1)}$ soft mode [see Figs. 2(a) and 2(c)], as the latter hardens upon warming, the two-phonon Raman signal also shifts its spectral weight to higher frequencies [see Figs. 3(a) and 3(b)]. Therefore, as suggested by previous thermal conductivity measurements[26] and supported by our findings, the coupling between acoustic and low-ω optical phonon branches appears to be a decisive ingredient for the low thermal conductivity of the Ta-based compounds.

The large difference between the κ values of the Ta- and Sb-based trirutiles[26], even though they have the same crystal structure, is remarkable. The strong anharmonic interactions revealed by the presence of a soft mode appear to be the root of this contrast. This is further evidenced by the significant broadening of high-ω phonons upon warming in the Ta-based compounds [see Figs. 5(b) and 5(d)]. For insulator compounds in the paramagnetic regime, phonon-phonon decay is the major source of line broadening and serves as an indirect tool to study the degree of anharmonicity in the crystal structure. Strong anharmonicity can be considered when phonon broadening deviates from the two-phonon decay model[32,37]. As shown in Figs. 5(b) and 5(d), the Sb-based compounds are well described by this model, while for the samples with Ta, the curve fails to reproduce the experimental data [see Figs. 5(b) and 5(d)]. Thus, as the thermal population of the $E_g^{(1)}$ soft mode becomes appreciable ($T \gtrsim 30$ K), the broadening of the high-ω modes deviates from the conventional anharmonic behavior found in Sb-based compounds. It is apparent that the slow dynamics of the low-ω $E_g^{(1)}$ soft mode are sensed by the faster high-ω modes as an effective structural disorder[51–54].

The association of the low thermal conductivity with a soft mode driven by anharmonicity opens the possibility to control κ by tuning the soft mode frequency through external parameters. In addition to pressure, as already commented, strain or chemical substitution are other effective tools for modifying physical properties. The latter is particularly relevant for thermoelectric materials, which, besides the low κ-values, should also present good electrical conductivity and large Seebeck coefficients. Thus, it would be interesting to evaluate whether the Ta-based trirutiles may become efficient thermoelectric materials under electronic doping, which also produces chemical pressure. If this is the case, the possible κ-tuning could boost the development of novel thermoelectric devices.

In summary, we conducted a detailed investigation of the lattice dynamics in trirutile compounds AB_2O_6 (A = Ni, Co, Zn; B = Sb, Ta) employing Raman spectroscopy measurements supported by DFT calculations. We demonstrate the existence of a low-ω phonon branch in ATa$_2$O$_6$ derived from a soft E_g mode. Its presence indicates an incipient instability in the crystal structure due to strong anharmonic contributions to the interatomic potentials. Moreover, we calculate and observe a high density of phonon states at low energies for the Ta-based trirutiles. The effective dynamical structural disorder induced by the thermal population of the soft mode appears to reduce the other phonons' lifetimes. These results not only provide insights into the low κ-values of Ta-based trirutiles in comparison to the Sb-based ones, but also indicate that compounds with phonon anharmonicity strong enough to induce zone-center soft modes is a promissing avenue for the discovery of novel thermoelectric materials.

SUPPLEMENTARY MATERIAL

See the supplementary material for detailed information on DFT calculations, sample growth, Raman spectroscopy measurements, symmetry analysis, and phonon assignments.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

R. Tartaglia: Conceptualization (supporting); Data Curation (supporting); Formal Analysis (lead); Investigation (lead), Writing – original draft (lead). **A. F. Lima:** Data Curation (lead); Investigation (supporting); Writing – original draft (supporting); **N. Prasai**: Investigation (equal). **A. B. Christian**: Investigation (equal). **J. J. Neumeier**: Supervision (equal); Writing – review & editing (supporting). **J. L. Cohn**: Supervision (equal); Writing – review & editing (supporting). **E. Granado**: Conceptualization (leading); Supervision (lead); Writing – review & editing (lead).

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available within the article.

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